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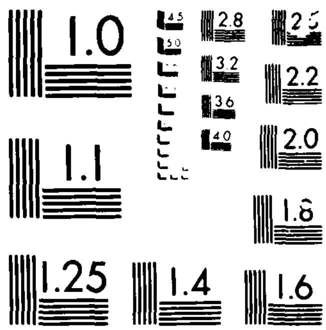

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Qualitative Results for Distributed Systems with Discrete & Stiffness with Application to Control

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Distributed parameter models of large flexible space structures subject to various control techniques have been studied. The main thrust has been to develop qualitative results which are independent of truncation or discretization approaches by treating the fully distributed model. Emphasis has been on controlling the transcient response of non-conservative linear partial differential equation models of such structures subject to a few point actuators.

Inequalities have been developed between the stiffness and damping operators which when satisfied guarantee that the response of a selfadjoint system will be uniformly exponentially stable. In addition, it has been shown that the inequalities insure that finite dimensional versions of the control problem converge to an optimal control of the fully distributed system subject to compact feedback as the number of modes in the finite model increases. The inequality developed constitues a generalization of the concept of under-damping normally used with single degree of freedom systems and provides a physical

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19 interpretation of the result.

The result listed for selfadjoint operators has been extended to non-selfadjoint systems in the special case that the coefficient operators have one selfadjoint factor and one symmetric factor such that the selfadjoint factors commute. Physical examples have been provided indicating that the extension is non trivial. This work has also resulted in a substantial parallel development for finite dimensional versions of the above.

Research Objectives

The research carried out under this grant [1] considered the vibration and control of a linear non-conservative dynamic systems described by partial differential equations of the form

$$u_{tt}(x,t) + L_1 u_t(x,t) + L_2 u(x,t) = f(x,t) \text{ on } \Omega \qquad (1)$$
$$Bu(x,t) = 0 \text{ on } \partial\Omega$$

where L_1 and L_2 are time invariant partial differential operators defined on a Hilbert Space, H, consisting of all appropriately smooth functions satisfying the boundary conditions, Ω is a bounded open region in R^n, n= 1,2,3, with boundary $\partial\Omega$, t denotes the time, and u(x,t) is the deflection at the position x in Ω. The subscript t denotes differentiation with respect to the time, t. The boundary conditions are represented by the time invariant partial differential operator B evaluated along the boundary $\partial\Omega$. Further assumptions are made to insure the existence of a solution of (1) of the form of a modal series.

The specific research objectives as listed in the original proposal (1) are:

(1) Extend the theory developed in [2-4] to include those problems which have non selfadjoint operators L_1 and L_2 but rather for which there exists an operator P defined on H such that the operators PL_i are selfadjoint.

(2) Use test functions to describe point actuators combined with the oscillation theorem of [2] to develop tighter bounds on the residual modes of the control problem and to extend these bounds to more general non conservative systems.

The next section describes the extent to which these objectives have been met.

Accomplishments

Objective (1) has been partially met by the following theorem, which will be presented at the 1985 CDC Meeting in December, which can be stated for (1) and is described in more detail in the attached abstracts and papers.

Theorem: If there exists a selfadjoint, positive definite operator P with compact resolvent such that the product operators PL_2 and PL_1 are non trival selfadjoint operators with compact resolvent, then the stability results and convergence results listed in [3] hold. Furthermore, the physical condition of underdamped modes provides a physical interpretation of the result by Gibson [3].

This result shows that modal control methods can be used in some non selfadjoint cases. The physical problem known as a Phlueger's column offers a non trival example of such a distributed parameter system.

Objective (2) has been more difficult to achieve and to date has only yielded results primarily for finite dimensional approximations. Of course the spirit of this work is to produce results based soley on the distributed formulation, so that this work is viewed as preliminary. Results obtained so far indicate that tight bounds can be found for the transcient response of an approximate system. The bounds are stated in terms of the relative values of the physical parameters of the system and can be shown to be better than existing theories when the eigenvalues of the stiffness operator are spread out and when the finite model has a large number of modes. This is encouraging for the fully distributed case.

In the special case that the stiffness and damping operators commute, simple, gross bounds can be found for the residual modes in the control problem and are given in [5], a copy of which is appended.

It has also been shown that the condition previously discovered by Gibson [3] can be derived from the underdampng condition given in [4]. This condition insures that optimal control laws derived based on finite dimensional models converge to optimal control for the full distributed system and that the response is uniformly exponentially stable. The result derived under this grant yields the physical interpretation that such systems are underdamped (but damped) systems . Necessary and sufficient conditions for the existence of "independent modal space control" are given in [7].

Publication List

The following lists those publications and the abstracts appearing in, or to appear in referred journals and proceedings representing work performed under this grant and acknowledged as such:

LIST 1 LUMPED PARAMETER SYSTEMS

1. Ahmadian, M. and Inman, D.J., "Some Stability Results for General Linear Lumped-Parameter Dynamic Systems", to appear in ASME Journal of Applied Mechanics.

ABSTRACT

A technique is presented for stability of equilibrium of general linear lumped-parameter dynamic systems. Two Liapunov functions are used to develop stability conditions which are directly in terms of the mass, damping, and stiffness matrices. The significance of what is presented here is twofold. First, it can be applied to general asymmetric systems. Second, it offers direct conditions which can easily be programmed on a digital computer to handle large-order systems. Many previously developed results, such as the KTC theorem anc its extensions, are mentioned. Next, it is shown that the present study may provide broader applications in the sense that it includes general systems and offers a more convenient approach. Examples are used to illustrate the validity and applications of the presented results.

2. Ahmadian, M. and Inman, D.J., "On the Stability of General Dynamic Systems Using a Liapunov's Direct Method Approach", to appear in the Journal of Computers and Structures.

ABSTRACT

A technique is presented for studying the stability of equilibrium of linear lumped-parameter systems involving general types of forces such as dissipative, non-conservative, gyroscopic, and circulatory. A modified approach to solving the Liapunov equation is used to provide a function V which is exploited to present different stability criteria for the equilibrium of such systems. It was shown that this approach is advantageous to the solution of the Liapunov equation, since it is more computationally attractive. Additionally, this may be used to determine the effects of different parameters on the systems stability or to design a controller for an actively controlled system. Several previously developed related works are studied and compared with this work. Furthermore, examples are used to illustrate the presented approach and some of its applications.

3. Ahmadian, M. and Inman, D.J., "Classical Normal Modes in Asymmetric Non Conservative Dynamic Systems", AIAA Journal, Vol. 22, No. 7, July 1984, pp 1012-1015.

ABSTRACT

The dynamic behavior of a general linear discrete system can be described by the vector differentialequation

$$Mx(t) + Cx(t) + Kx(t) = 0 \tag{1}$$

where M,C, and K are mass, damping, and stiffness matrices, respectively. The usual treatment of these systems assumes that Eq. (1) is symmetric. Although this assumption is justified for passive systems, in many problems of interest in aeronautics, ship vibrations and active control of large space structures, Eq. (1) cannot be presented in a symmetric form This motivates the study of the behavior and properties of this class of problems and also ar attempt to derive relations similar to those of symmetric systems.

4. Ahmadian, M. and Inman, D.J., "On the Nature of Eigenvalues of General Non Conservaative Systems", in Journal of Applied Mechanics, Vol. 51, No. 1, March 1984, pp. 193-194.

ABSTRACT

The dynamic behavior of a general lumped-parameter system can be described by the vector differentialequation

$$Mx(t) + Cx(t) + Kx(t) = 0 \tag{1}$$

where M,C, and K are referred to as mass, damping, and stiffness matrices, and x(t) represents the displacement vector. For passive systems of the conventional type, the matrices M, C, and K are symmetric positive-definite or semidefinite and a number of well-known theories have been developed for such systems. One such theory is that developed by Inman and Andry on the oscillatory behavior of a symmetric system [1]. It has been shown that similar to a single degree-of-freedom system the oscillatory nature of a symmetric multidegree-of-freedom system can be predicted by forming a critical damping matrix (i.e., $C_{cr} = 2(M^{-1/2}KM^{-1/2})^{1/2}$) and studying the definiteness of the matrix $M^{-1/2}CM^{-1/2} - C_{cr}$. Here the superscript - 1/2 refers to the inverse of the positive defnite square root, a positive-definite matrix.

5. Inman, D.J., "Dynamics of Asymmetric Non-Conservative Systems", Journal of Applied Mechanics, Vol. 50, No. 1, 1983, pp. 199-203.

ABSTRACT

Recently several authors [1-4] have considered the vibrations of asymmetric dynamical systems which may be successfully modeled by a vector differential equation of the form

$$A\ddot{x}(t) + B\dot{x}(t) + Cx(t) = f(t) \qquad (1)$$

where A, B, and C are general real n x n matrices (lacking any particular symmetry or definiteness) and where x(t) and f(t) are n-dimensional column vectors of generalized coordinates and forces, respectively. Problems of this type arise in vehicle dynamics as well as in the study of circulatory systems and, as pointed out by Fawzy and Bishop [4] have not been properly catered to.

6. Ahmadian, M. and Inman, D.J., "Closed Loop Stability of Large Space Structures with Reduced Order Controllers", to appear in the Proceedings of the 5th VPI and SU Symposium Dynamics and Control of Large Structures.

ABSTRACT

Modal truncation in the dynamic models of large space structures can cause well-known stability problems when these models are used as the design basis for active structural control. In other words, the asymptotic stability of a full-order closed-loop system cannot necessarily be ensured by the asymptotic stability of the reduced-order model. The main goal of the present study is to develop conditions which will ensure asymptotic stability of the full-order closed-loop system, when the reduced-order closed-loop system is asymptotically stable. A finite dimensional model of large space structures are exploited here to provide such conditions. These conditions are in terms of the modeled and residual submatrices of the coefficient matrices and may serve as a means of improving, or guiding, the design of a reduced-order controller.

7. DeCaro, S.M., and Inman, D.J., "Eigenvalue Placement and Stabilization by Constrained Optimization", Proceedings of the JPL Workshop on Identification and Control of Flexible Space Structures.

ABSTRACT

A pole placement algorithm is proposed which uses constrained non-linear optimization techniques on a finite dimensional model of a linear n degree of freedom system. Low order feedback control is assumed where r poles may be assigned; r being the rank of the sensor coefficient matrix. It is shown that by combining feedback control theory methods with optimization techniques, one can ensure the stability characteristics of a system, and can alter its transient response.

8. Ahmadian, M. and Inman, D.J., "Modal Analysis in Non-Conservative Dynamics Systems", Proceedings of 2nd International Conference on Modal Analysis, Vol. I, February 1984, pp. 340-344.

ABSTRACT

This work examines the existence and use of classical normal modes in the modal analysis of general non-conservative structures. The use of theoretical modal analysis has received increased attention in recent years with the advent and use of flexible structures both in the fields of large structure control and the control and design of flexible link robots. The problems addressed here are those resulting from systems which can be modeled by second order linear vector differential equations with constant coefficient matrices. Specifically, conditions for the existence of classical normal modes in various classes of problems as given by previous authors are compared, along with methods of calculating the appropriate modal coordinates. In addition, modal analysis techniques for those problems which do not possess normal modes (i.e., systems with quasi-normal modes) are discussed.

9. Inman, D.J. and Hsieh, C.I., "Controllability of Non-Self Adjoint Flexible Systems", ASME Paper #83-WA/DSC-16, Winter Annual Meeting, November 1983.

ABSTRACT

The special structure of a symmetrizable asymmetric matrix is used to study the controllability and observability of finite dimensional modes of non self-adjoint flexible structures. The conditions are relatively simple to apply and require less computation than using the standard controllability matrix. The method is illustrated by an example.

LIST 2 DISTRIBUTED PARAMETER SYSTEMS

1. Inman, D.J., "Modal Decoupling Conditions for Distributed Control of Flexible Structures", AIAA Journal of Guidance Control and Dynamics, July-August 1984.

ABSTRACT

The problem of the control of distributed parameter systems can be roughly divided into twc approaches;
1) discretize the system in space and then use finite dimensional control theory; and 2) deal with the distributed model directly without discretizing. Recently, Meirovitch and Baruh; proposed a scheme for the optimal control of a certain class of conservative distributed parameter systems without resorting to discretization. In particular, they treated the control of self-adjoint conservative systems having known eigensolutions. It is the intent of this Note to point out that their results are applicable to a more general class of problems that includes nonconservative forces and to note that the necessary conditions are available for the existence of decoupling control laws. Decoupling control laws are defined to be those control laws dependent only on the model state vector of the decoupled equation. This yields an infinite set of independent equations including the feedback control.

2. Inman, D.J., "Symmetrizable Structures and Modal Control", to appear in the Proceedings of the Second Symposium on Structural Control".

ABSTRACT

This work examines the extent to which modal expansions exist for linear dynamic systems described by partial differential equations defined on a Hilbert space. Modal expansions, as used here, refer to the existence of a complete orthonormal set of functions associated witr the operators defining the class of systems considered. It is shown that a certain class of non-self-adjoint operators can be defined as being self-adjoint with respect to a given self-adjoint operator. These product operators are then used to define eigenfunction expansions and to exmine standard modal control methods. A standard example of position and velocity feedback control of a non-self-adjoint distributed structure is given (a damped generalized Pfluger's column).

3. Inman, D.J., "Finite Control of Undamped Distributed Parameter Systems", Proceedings of the JPL Workshop on Identification and Control of Flexible Space Structures.

ABSTRACT

In the development of the theory of control for large flexible space structures, two important questions have been raised about the effect of using finite dimensional controls on inherently distributed parameter structures. The first question raised [1] has come to be known as control spillover [2]. Spillover can roughly be defined as the effect of energy addec to unmodeled modes of a structure by the action of control laws derived from information about the modeled modes only. More recently, the question of whether or not a control law based on finite dimensional approximations of the distributed system will converge to a control law which is optimal for the full distributed parameter model has been raised. Gibson [3] has shown that the answer to this question is yes if enough damping is modeled.

The result presented here, shows that both of these problems i.e. spillover and convergence, are manageable when the distributed parameter system under consideration is underdamped [4-5].

4. Inman, D.J., "Critical Damping Complex Structures", to appear in the Proceedings of the Air Force Vibration Damping Workshop, August 1984.

ABSTRACT

This work examines the concept of critical damping normally defined for single degree of freedom systems as applied to more complex models of structures and their control systems. Here, complex models refers to lumped mass models such as finite element models (FEM), full distributed parameter (DPS) models (described by sets of partial differential equations), hybrid systems (systems with lumped and distributed parameter elements, such as passive control systems) and active control systems consisting of the above mentioned models subject to position and velocity feedback. Recent work on critical damping of FEM and DPS models is reviewed and applied to more general linear models.

5. Ng, C.K. and Inman, D.J., "Active Control of Decoupled Underdamped Systems", Proceedings of the 25th Structures Dynamics and Materials Conference, May 1984, pp. 192-200.

ABSTRACT

The implementation of a state feedback controller for a class of underdamped distributed parameter systems (DPS) possessing classical normal modes is studied. In this control scheme, a continuous system in time and space is reduced and decoupled into a sufficiently large but finite system described by its time dependent harmonic or modal amplitudes. This model is then divided into two parts, the controlled and the residual (uncontrolled) subsystems. For the controlled subsystem, controllability and observability conditions are derived. It is demonstrated that, in theory, only one actuator and one sensor are required for controllability and observability. In this case, these controlability and observability conditions are expressed simply as functions of eigenfunctions and eigenvalues of the DPS. The authors note that these results on controllability and observability are consistent with the results on more restricted models obtained by previous investigators. Upper bounds for the residual modal amplitudes of the DPS have been derived. These bounds show that the control spillover does not destabilize the residual subsystem.

Associated Professional Personnel

The following is a list of graduate students, degree (degree sought), thesis title, and date awarded (excepted) for those student advised by the principal investigator during this grant period (July 1, 1983 to June 30, 1984) who worked on research related to the grant. Those students who received some support from this grant are indicated by an asterisk. (all are U.S, citizens, except Ahmadian)

Musiol, Irene, "Bounds on the Forced Response of Distributed Parameter Systems", M.S. Project, March 1984.

Schultz, Mark, "Study of Damper Placement for Beam Vibration Control", M.S. Project, March 1984.

DeCaro, Sandra*, "Eigenvalue Assignment for Non-Conservative Dynamic Systems", M.S. Thesis, Marcy 1984.

Ahmadian, Mehdi*, "Dynamics and Control of Asymmetric Systems", Ph.D. Dissertation, July 1984.

Yae, Kwang*, "Estimates of Decay Rates for Underdamped Distributed Parameter Systems", Ph.D., Expected, 1986.

Zimmerman, David, "Digital Control of the NASA-UVS Proof Mass Actuator", M.S. Thesis, April 1984.

Zimmerman, David, "Reliability Problems in Large Flexible Space Structures", Ph.D. expected 1986.

Ghamine, George, "Numerical Solutions of Asymmetric Systems", M.S. Project, April 1984.

Cudney, Harley, "Digital Control of a Beam with Joint", M.S. expected 1985.

Ebbing, David, "Damping in Composite Materials", M.S. Thesis expected 1984-85.

Hendrickson, William, "Normal Modes in Composite Materials", M.S. Thesis 1985.

Duke, Patricia, "Dynamic Stiffness of Control System", M.S. Thesis expected 1985.

Wan, K.W., "Control of Systems with Temperature Dependent Parameters", Ph.D. expected 1988.

Tylock, James, "Noise Control Experiments", M.S. Thesis expected September 1985.

Lee Glauser, Gina, "Experimental Verification of ERA", M.S. Thesis expected December 1985.

Belos, John, "Non-Normal Mode Systems", M.S. Thesis expected December 1985.

Interactions

The following is a list of presentations made or to be made by the principle investigator at conferences, meetings, and seminars representing work performed under the grant during the period from July 1, 1982 to June 30, 1985.

"Control and Vibrations of Non-Selfadjoint Distributed Parameter Systems", Seminar Series Department of Electrical Engineering, Univeristy of Rochester, October 1983.

"Modal Control of a Class of Non-Selfadjoint Systems", SIAM 1983 Fall Meeting, Norfolk, Virginia, November 1983.

"Controllability of Non-Selfadjoint Flexible Systems", ASME 1983 Winter Annual Meeting, Boston, Massachusetts, November 1983.

"Vibration and Control of Large Flexible Space Structures", Seminar Series Department of Mechanical Engineering, Ohio State University, January 1984.

"Vibration and Control of Space Structures", Western New York High Technology Lecture Series, Bufalo, New York, January 1984.

"Modal Analysis in Non-Conservative Dynamic Systems", 2nd International Conference on Modal Analysis, Orlando, Florida, 1984.

"Critical Damping in Complex Structure", Air Force Vibration Damping Workshop, Long Beach, California, February 1984.

"Active Control of Decoupled Underdamped Systems", 25th Structures Dynamics and Materials Conference, Palm Spings, California, May 1984.

"Microprocessor Control of the NASA-UVA Proof Mass Actuator", AIAA Dynamics Specialist Conference, Palm Springs, California, May 1984.

"Frequency Domain Analysis of a Plate with Discrete Elements Attached", Dynamics Specialist Conference, Palm Springs, California, May 1984.

"Eigenvalue Assignment by Constrained Optimization", JPL Workshop on Identification and Control of Flexible Space Structures, San Diego, California, June 1984.

"Finite Control in Underdamped Distributed Parameter Systems", JPL Workshop on Identification and Control of Flexible Space Structures, San Diego, California, June 1984.

"Research in Distributed Parameter Control Theory", AFOSR Forum on Large Space Structures, McLean, Virignia, June 1984.

"Modes and Critical Damping in Asymmetric Linear Dynamic Systems", XVI ITACM, Lyngby, Denmark, August 1984.

"Decay Rates for Linear Dynamical Systems", Invited, 21st Society of Engineering Science Meeting, Blacksburg, Virginia, October 1983.

"Flexible Space Structure Control", Seminar Series, Clemson University, Clemson,South Carolina, October 1984.

"Control of Large Space Structures", System Theory Seminar, University of Minnesota, Minneapolis, Minnesota, November 1984.

"Controllability and Observability of Flexible Gyroscopic Systems" 21st Annual Society of Engineering Science Meeting, October 1984.

"On the Stability of General Linear Dynamic Systems" 21st Annual Society of Engineering Science Meeting, October 1984.

"Controlling Flexible Structures", Seminar Series, Stevens Institute of Technology, Hobogen, New Jersey, November 1984.

"Definition of a Damping Ratio Matrix", 3rd International Conference on Modal Analysis, January 1985.

"Response Bounds for Damped Linear Systems", 26th Structures, Dynamics and Materials Conference, April 1985.

"Non-Selfadjoint System", ICASE, NASA Langley Research Center, May 1985.

"Control of Symmetrizable Systems, Seminar Series, University of Waterloo, Canada, June 1985.

"Dynamics and Control of Temperature Dependent Flexible Structures", AFOSR Third Forum on Large Space Structures, July 1985.

"Modal Control of Symmetrizable Structures", 2nd International Symposium on Structural Control, July 1985.

"Identification of a Damping Matrix from Modal Data", 5th VPI and SU Symposium on Dynamics and Control of Large Structures, June 1985.

References

[1] Inman, D.J., "Qualitative Results for Distributed Systems with Discrete Damping and Stiffness with Applicaation to Control", AFOSR Grant #820242, January 1984 (and December 1983).

[2] Inman, D.J. and Andry, A.N., Jr., "The Nature of the Temporal Solutions of Damped Linear Distributed Parameter Systems with Normal Modes", Journal of Applied Mechanics, Vol. 49, 1982, pp. 867-870.

[3] Gibson, J.S., "An Analysis of Optimal Modal Regulation: Convergence and Stability", SIAM Journal of Control and Optimization, Vol. 19, 1981, pp. 686-706.

[4] Inman, D.J., "Oscillatory Damped Distributed Parameter System", Mechanics Research Communications, Vol. 9, No. 2, 1982, pp. 101-107.

[5] Ng, C.K. and Inman, D.J., "Active Control of Decoupled Underdamped Systems", Proceedings of the AIAA 25th SIAM Conference Part II, 1984, pp. 192-201.

[6] Inman, D.J., "Finite Control in Underdamped Distributed Parameter Systems", JPL Workshop on Identification and Control of Flexible Space Structures Proceedings, 1984.

[7] Inman, D.J., "Modal Decoupling Conditions for Distributed Control of Flexible Structures", AIAA Journal of Guidance, Control and Dynamics, Vol. 7, No. 6, November-December 1984, pp. 750-752.

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